CORRIGENDUM TO "A PRIORI BOUNDS FOR WEAK SOLUTIONS TO ELLIPTIC EQUATIONS WITH NONSTANDARD GROWTH" [DISCRETE CONTIN. DYN. SYST. SER. S 5 (2012), 865–878.]

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In this corrigendum we correct a lemma concerning the geometric convergence of sequences of numbers which was used in [2] as Lemma 2.1. As a consequence the statement in the main result changes a bit and the corresponding proof needs some minor different arguments to be fitted.

(a) First, we replace Theorem 1.1 in [2] by the following one:

**Theorem 1.1.** Let the assumptions in (H) be satisfied. Then there exist positive constants \( \alpha = \alpha(p,q_0,q_1) \) and \( C = C(p,q_0,q_1,a_3,a_4,a_5,b_0,b_1,b_2,c_0,c_1,N,\Omega) \) such that the following assertions hold.

(i) If \( u \in W^{1,p}(\Omega) \) is a weak subsolution of (1.1), then both \( \text{ess sup}_\Omega u \) and \( \text{ess sup}_\Gamma u \) are bounded from above by

\[
C \left[ 1 + \int_\Omega u^{q_0}(x) \, dx + \int_\Gamma u^{q_1}(x) \, d\sigma \right]^\alpha.
\]

(ii) If \( u \in W^{1,p}(\Omega) \) is a weak supersolution of (1.1), then both \( \text{ess inf}_\Omega u \) and \( \text{ess inf}_\Gamma u \) are bounded from below by

\[
-C \left[ 1 + \int_\Omega (-u)^{q_0}(x) \, dx + \int_\Gamma (-u)^{q_1}(x) \, d\sigma \right]^\alpha.
\]

(b) Next, we replace Corollary 1.2 in [2] by the following one:

**Corollary 1.2.** Let the assumptions (H) be satisfied and let \( u \in W^{1,p}(\Omega) \) be a weak solution of (1.1). Then \( u \in L^\infty(\Omega), L^\infty(\Gamma) \) and the estimates in (i) and (ii) from Theorem 1.1 are valid.

(c) Replace reference [32] on page 4, line 5 from bottom by the new reference [1].

(d) Now, we replace Lemma 2.1 in [2] by the following one:

**Lemma 2.1.** Let \( \{Y_n\}, n = 0, 1, 2, \ldots, \) be a sequence of positive numbers, satisfying the recursion inequality

\[
Y_{n+1} \leq Kb^n \left( Y_n^{1+\delta_1} + Y_n^{1+\delta_2} \right), \quad n = 0, 1, 2, \ldots,
\]

for some \( b > 1, K > 0 \) and \( \delta_2 \geq \delta_1 > 0 \). If

\[
Y_0 \leq \min \left( 1, (2K)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1}} \right)
\]

or

\[
Y_0 \leq \min \left( (2K)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1}}, (2K)^{-\frac{1}{\delta_2}} b^{-\frac{1}{\delta_2}} - \frac{\delta_2 - \delta_1}{\delta_1} \right),
\]

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then $Y_n \leq 1$ for some $n \in \mathbb{N} \cup \{0\}$. Moreover,

$$Y_n \leq \min \left( 1, (2K)^{-\frac{1}{\gamma b-\frac{1}{\gamma} b_{1/2}}} \right), \quad \text{for all } n \geq n_0,$$

where $n_0$ is the smallest $n \in \mathbb{N} \cup \{0\}$ satisfying $Y_n \leq 1$. In particular, $Y_n \to 0$ as $n \to \infty$.

We note that Lemma 2.1 stated in [2] would have been correct if $K > 1$ instead of $K > 0$. However, we need in our treatment such a result for arbitrary positive $K$.

Now, at two places in the proof of Theorem 1.1, we need some minor changes.

(e) On page 8, after line 3, we add the following paragraph:

“Here, $(p_i^-)$ and $(p_i^+)$ are defined by, for all $i = 1, \ldots, m$,

$$(p_i^-)_* = \begin{cases} N(p_i^-) & \text{if } (p_i^-) < N, \\ q_i^- + 1 & \text{if } (p_i^-) \geq N, \end{cases} \quad (p_i^+)_* = \begin{cases} (N-1)(p_i^-) & \text{if } (p_i^-) < N, \\ q_i^+ + 1 & \text{if } (p_i^-) \geq N, \end{cases}$$

where $q_i^- = \max_{x \in \Gamma} q_0(x)$ and $q_i^+ = \max_{x \in \Gamma} q_1(x)$ (see Section 2).”

(f) Replace the paragraph on page 12 from formula (3.23) until line 4 from bottom by the following paragraph:

$$Y_0 = \int_{\Omega} (u - k)^{q_0(x)} dx + \int_{\Gamma} (u - k)^{q_1(x)} d\sigma$$

$$\leq \min \left[ \left( \frac{16K}{k_0 (1-H)} \right)^{\frac{1}{\gamma_b}} b^{\frac{1}{\gamma_b}} \left( \frac{16K}{k_0 (1-H)} \right)^{-\frac{1}{\gamma_b}} b^{-\frac{1}{\gamma_b}} \frac{\hat{\eta}}{\hat{\eta}{\gamma_b}} - \frac{\epsilon_2 - \epsilon_1}{\gamma_b}, \left( \frac{16K}{k_0 (1-H)} \right)^{-\frac{1}{\gamma_b}} b^{-\frac{1}{\gamma_b}} \frac{\hat{\eta}}{\hat{\eta}{\gamma_b}} - \frac{\epsilon_2 - \epsilon_1}{\gamma_b} \right].$$

Relation (3.23) is clearly satisfied if

$$\int_{\Omega} u_+^{q_0(x)} dx + \int_{\Gamma} u_+^{q_1(x)} d\sigma$$

$$\leq \min \left[ \left( \frac{16K}{k_0 (1-H)} \right)^{-\frac{1}{\gamma_b}} b^{\frac{1}{\gamma_b}} \left( \frac{16K}{k_0 (1-H)} \right)^{-\frac{1}{\gamma_b}} b^{-\frac{1}{\gamma_b}} \frac{\hat{\eta}}{\hat{\eta}{\gamma_b}} - \frac{\epsilon_2 - \epsilon_1}{\gamma_b} \right].$$

Hence, if we choose $k$ such that

$$k = \left( 1 + \frac{1}{k_0 (1-H)} b^{\frac{1}{\gamma_b}} \frac{1}{\gamma_b} b^{\frac{1}{\gamma_b}} + \frac{\epsilon_2 - \epsilon_1}{\gamma_b} \right) \times \left( 1 + \int_{\Omega} u_+^{q_0(x)} dx + \int_{\Gamma} u_+^{q_1(x)} d\sigma \right),$$

then (3.24) and in particular (3.23) are satisfied. Since $k_n \to 2k$ as $n \to \infty$ we obtain

$$\text{ess sup}_\Omega u \leq 2k \quad \text{and} \quad \text{ess sup}_\Gamma u \leq 2k$$

with $k$ given in (3.25).”
References


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