CORRIGENDUM TO "A PRIORI BOUNDS FOR WEAK SOLUTIONS TO ELLIPTIC EQUATIONS WITH NONSTANDARD GROWTH" [DISCRETE CONTIN. DYN. SYST. SER. S 5 (2012), 865–878.]

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In this corrigendum we correct a lemma concerning the geometric convergence of sequences of numbers which was used in [2] as Lemma 2.1. As a consequence the statement in the main result changes a bit and the corresponding proof needs some minor different arguments to be fitted.

(a) First, we replace Theorem 1.1 in [2] by the following one:

Theorem 1.1. *Let the assumptions in (H) be satisfied. Then there exist positive constants* $\alpha = \alpha(p, q_0, q_1)$ *and* $C = C(p, q_0, q_1, a_3, a_4, a_5, b_0, b_1, b_2, c_0, c_1, N, \Omega)$ *such that the following assertions hold.*

(i) *If* $u \in W^{1,p(\cdot)}(\Omega)$ *is a weak subsolution of (1.1), then both* $\operatorname{ess\,sup}_\Omega u$ *and* $\operatorname{ess\,sup}_\Gamma u$ *are bounded from above by*

$$C\left[1 + \int_\Omega u_+^{q_0(x)} dx + \int_\Gamma u_+^{q_1(x)} d\sigma\right]^\alpha.$$

(ii) *If* $u \in W^{1,p(\cdot)}(\Omega)$ *is a weak supersolution of (1.1), then both* $\operatorname{ess\,inf}_\Omega u$ *and* $\operatorname{ess\,inf}_\Gamma u$ *are bounded from below by*

$$-C\left[1 + \int_\Omega (-u)_+^{q_0(x)} dx + \int_\Gamma (-u)_+^{q_1(x)} d\sigma\right]^\alpha.$$

(b) Next, we replace Corollary 1.2 in [2] by the following one:

Corollary 1.2. *Let the assumptions (H) be satisfied and let* $u \in W^{1,p(\cdot)}(\Omega)$ *be a weak solution of (1.1). Then* $u \in L^\infty(\Omega), L^\infty(\Gamma)$ *and the estimates in (i) and (ii) from Theorem 1.1 are valid.*

(c) Replace reference [32] on page 4, line 5 from bottom by the new reference [1].

(d) Now, we replace Lemma 2.1 in [2] by the following one:

Lemma 2.1. *Let* $\{Y_n\}, n = 0, 1, 2, \dots,$ *be a sequence of positive numbers, satisfying the recursion inequality*

$$Y_{n+1} \le K b^n \left(Y_n^{1+\delta_1} + Y_n^{1+\delta_2}\right), \quad n = 0, 1, 2, \dots,$$

for some $b > 1$, $K > 0$ *and* $\delta_2 \ge \delta_1 > 0$. *If*

$$Y_0 \le \min\left(1, (2K)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1^2}}\right)$$

or

$$Y_0 \le \min\left((2K)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1^2}}, (2K)^{-\frac{1}{\delta_2}} b^{-\frac{1}{\delta_1\delta_2} - \frac{\delta_2-\delta_1}{\delta_2^2}}\right),$$

then $Y_n \leq 1$ *for some* $n \in \mathbb{N} \cup \{0\}$*. Moreover,*

$$Y_n \leq \min\left(1, (2K)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1^2}} b^{-\frac{n}{\delta_1}}\right), \qquad \text{for all } n \geq n_0,$$

where n_0 *is the smallest* $n \in \mathbb{N} \cup \{0\}$ *satisfying* $Y_n \leq 1$*. In particular,* $Y_n \to 0$ *as* $n \to \infty$*.*

We note that Lemma 2.1 stated in [2] would have been correct if $K > 1$ instead of $K > 0$. However, we need in our treatment such a result for arbitrary positive K.

Now, at two places in the proof of Theorem 1.1, we need some minor changes.

(e) On page 8, after line 3, we add the following paragraph:

"Here, $\left(p_i^-\right)^*$ and $\left(p_i^-\right)_*$ are defined by, for all $i = 1, \ldots, m$,

$$\left(p_i^-\right)^* = \begin{cases} \frac{N\left(p_i^-\right)}{N-\left(p_i^-\right)} & \text{if } \left(p_i^-\right) < N, \\ q_0^+ + 1 & \text{if } \left(p_i^-\right) \geq N, \end{cases} \qquad \left(p_i^-\right)_* = \begin{cases} \frac{(N-1)\left(p_i^-\right)}{N-\left(p_i^-\right)} & \text{if } \left(p_i^-\right) < N, \\ q_1^+ + 1 & \text{if } \left(p_i^-\right) \geq N, \end{cases}$$

where $q_0^+ = \max_{x\in\overline{\Omega}} q_0(x)$ and $q_1^+ = \max_{x\in\Gamma} q_1(x)$ (see Section 2)."

(f) Replace the paragraph on page 12 from formula (3.23) until line 4 from bottom by the following paragraph:

"

$$\begin{aligned} Y_0 &= \int_\Omega (u-k)_+^{q_0(x)} dx + \int_\Gamma (u-k)_+^{q_1(x)} d\sigma \\ &\leq \min\left[\left(\frac{16K}{k^{q_0^-(1-\hat{\eta})}}\right)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1^2}}, \left(\frac{16K}{k^{q_0^-(1-\hat{\eta})}}\right)^{-\frac{1}{\delta_2}} b^{-\frac{1}{\delta_1\delta_2}-\frac{\delta_2-\delta_1}{\delta_2^2}}\right]. \end{aligned} \tag{3.23}$$

Relation (3.23) is clearly satisfied if

$$\begin{aligned} &\int_\Omega u_+^{q_0(x)} dx + \int_\Gamma u_+^{q_1(x)} d\sigma \\ &\leq \min\left[\left(\frac{16K}{k^{q_0^-(1-\hat{\eta})}}\right)^{-\frac{1}{\delta_1}} b^{-\frac{1}{\delta_1^2}}, \left(\frac{16K}{k^{q_0^-(1-\hat{\eta})}}\right)^{-\frac{1}{\delta_2}} b^{-\frac{1}{\delta_1\delta_2}-\frac{\delta_2-\delta_1}{\delta_2^2}}\right]. \end{aligned} \tag{3.24}$$

Hence, if we choose k such that

$$\begin{aligned} k = &\left(1 + (16K)^{\frac{1}{q_0^-(1-\hat{\eta})}} b^{\frac{1}{\delta_1 q_0^-(1-\hat{\eta})} + \frac{\delta_2-\delta_1}{\delta_2 q_0^-(1-\hat{\eta})}}\right) \\ &\times \left(1 + \int_\Omega u_+^{q_0(x)} dx + \int_\Gamma u_+^{q_1(x)} d\sigma\right)^{\frac{\delta_2}{q_0^-(1-\hat{\eta})}}, \end{aligned} \tag{3.25}$$

then (3.24) and in particular (3.23) are satisfied. Since $k_n \to 2k$ as $n \to \infty$ we obtain

$$\operatorname*{ess\,sup}_\Omega u \leq 2k \quad \text{and} \quad \operatorname*{ess\,sup}_\Gamma u \leq 2k$$

with k given in (3.25). "

References

[1] K. Ho, I. Sim, *Corrigendum to "Existence and some properties of solutions for degenerate elliptic equations with exponent variable"[Nonlinear Anal. 98 (2014), 146–164]*, Nonlinear Anal. **128** (2015), 423–426.

[2] P. Winkert, R. Zacher, *A priori bounds for weak solutions to elliptic equations with nonstandard growth*, Discrete Contin. Dyn. Syst. Ser. S 5 **4** (2012), 865–878.

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