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Corrigendum

Corrigendum to “On a quasilinear elliptic problem with convection term and nonlinear boundary condition” [Nonlinear Anal. 187 (2019) 159–169]

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ABSTRACT

We correct the proof of Theorem 4.6 in “On a quasilinear elliptic problem with convection term and nonlinear boundary condition” [Nonlinear Anal. 187 (2019) 159–169].

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Since inequality (4.13) in [1] is not true in general, the proof of Theorem 4.6 has to be amended. Accordingly, we need to change condition (U1) while (U2) remains the same. The assumptions read as follows.

(U1) There exist $c_1, c_2, c_3 \in \mathbb{R}_+$ such that $c_2 > c_3$ and

$$(f(x,s,\xi) - f(x,t,\xi))(s-t) \le c_1|s-t|^p \quad \forall x \in \Omega,\ s,t \in \mathbb{R},\ \xi \in \mathbb{R}^N,$$
$$(g(x,s) - g(x,t))(s-t) \le c_2|s-t|^p - c_3|s-t|^2 \quad \forall x \in \partial\Omega,\ s,t \in \mathbb{R}.$$

(U2) With appropriate $\rho \in L^{r'}(\Omega)$, where $1 < r' < p^*$, and $c_4 \in \mathbb{R}_+$ one has both $\xi \mapsto f(x,s,\xi) - \rho(x)$ linear for every $(x,s) \in \Omega \times \mathbb{R}$ and

$$|f(x,s,\xi) - \rho(x)| \le c_4|\xi| \quad \text{in} \quad \Omega \times \mathbb{R} \times \mathbb{R}^N.$$

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We can now formulate our uniqueness result.

Theorem 4.6. *Let (H), (U1), and (U2) be satisfied.*

(a) If $p := 2 > q > 1$ *and*

$$\max\left\{c_1, \frac{c_2 - c_3}{\zeta}\right\} + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}} < 1 \tag{4.7}$$

then (P_μ) *admits a unique weak solution for every* $\mu > 0$.

(b) If $p > q := 2$, *then* (P_μ) *possesses only one weak solution provided*

$$\max\left\{c_1, \frac{c_2}{\zeta}\right\} < 2^{2-p} \quad \text{and} \quad \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}} < \min\left\{\mu, \frac{c_3}{\zeta}\right\}. \tag{4.8}$$

Proof. Fix $\mu > 0$. Theorem 4.1 gives a weak solution $u_\mu \in W^{1,p}(\Omega) \cap L^\infty(\Omega)$ of (P_μ). Suppose $v_\mu \in W^{1,p}(\Omega)$ enjoys the same property. Using (3.7) with $\varphi := u_\mu - v_\mu$ easily leads to

$$\begin{aligned}
&\langle A_p(u_\mu) - A_p(v_\mu), u_\mu - v_\mu\rangle + \mu\langle A_q(u_\mu) - A_q(v_\mu), u_\mu - v_\mu\rangle \\
&\quad + \zeta \int_{\partial\Omega} (|u_\mu|^{p-2}u_\mu - |v_\mu|^{p-2}v_\mu)(u_\mu - v_\mu)\,d\sigma \\
&= \int_\Omega (f(x, u_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla u_\mu))(u_\mu - v_\mu)\,dx \\
&\quad + \int_\Omega (f(x, v_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla v_\mu))(u_\mu - v_\mu)\,dx \\
&\quad + \int_{\partial\Omega} (g(x, u_\mu) - g(x, v_\mu))(u_\mu - v_\mu)\,d\sigma.
\end{aligned} \tag{4.9}$$

(a) Let $p := 2 > q > 1$. By monotonicity of A_q, the left-hand side in (4.9) can be estimated through

$$\begin{aligned}
&\langle A_2(u_\mu) - A_2(v_\mu), u_\mu - v_\mu\rangle + \mu\langle A_q(u_\mu) - A_q(v_\mu), u_\mu - v_\mu\rangle \\
&\quad + \zeta \int_{\partial\Omega} (u_\mu - v_\mu)(u_\mu - v_\mu)\,d\sigma \\
&\geq \|\nabla(u_\mu - v_\mu)\|_2^2 + \zeta\|u_\mu - v_\mu\|_{2,\partial\Omega}^2 = \|u_\mu - v_\mu\|_{\zeta,2}^2,
\end{aligned} \tag{4.10}$$

where $\|\cdot\|_{\zeta,2}$ denotes the equivalent norm (2.1). As regards the right-hand side, due to (U1), (U2), Hölder's inequality, and (3.14), we have

$$\begin{aligned}
&\int_\Omega (f(x, u_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla u_\mu))(u_\mu - v_\mu)\,dx \\
&\quad + \int_\Omega (f(x, v_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla v_\mu))(u_\mu - v_\mu)\,dx \\
&\quad + \int_{\partial\Omega} (g(x, u_\mu) - g(x, v_\mu))(u_\mu - v_\mu)\,d\sigma \\
&\leq c_1\|u_\mu - v_\mu\|_2^2 + \int_\Omega \left(f\left(x, v_\mu, \nabla\left(\frac{1}{2}(u_\mu - v_\mu)^2\right)\right) - \rho(x)\right)dx \\
&\quad + c_2\|u_\mu - v_\mu\|_{2,\partial\Omega}^2 - c_3\|u_\mu - v_\mu\|_{2,\partial\Omega}^2 \\
&\leq \max\left\{c_1, \frac{c_2 - c_3}{\zeta}\right\}\|u_\mu - v_\mu\|_{\zeta,2}^2 + c_4\int_\Omega |u_\mu - v_\mu||\nabla(u_\mu - v_\mu)|\,dx \\
&\leq \left(\max\left\{c_1, \frac{c_2 - c_3}{\zeta}\right\} + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}}\right)\|u_\mu - v_\mu\|_{\zeta,2}^2.
\end{aligned} \tag{4.11}$$

Gathering (4.9)–(4.11) together now yields

$$\|u_\mu - v_\mu\|_{\zeta,2}^2 \le \left(\max\left\{c_1, \frac{c_2 - c_3}{\zeta}\right\} + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}}\right) \|u_\mu - v_\mu\|_{\zeta,2}^2,$$

which implies $u_\mu = v_\mu$ because of (4.7).

(b) Let $p > q := 2$. Likewise before, the left-hand side of (4.9) becomes

$$\begin{aligned}
&\langle A_p(u_\mu) - A_p(v_\mu), u_\mu - v_\mu\rangle + \mu\langle A_2(u_\mu) - A_2(v_\mu), u_\mu - v_\mu\rangle \\
&\quad + \zeta \int_{\partial\Omega} \left(|u_\mu|^{p-2}u_\mu - |v_\mu|^{p-2}v_\mu\right)(u_\mu - v_\mu)\, d\sigma \\
&\ge 2^{2-p}\|\nabla(u_\mu - v_\mu)\|_p^p + \mu\|\nabla(u_\mu - v_\mu)\|_2^2 \\
&\quad + \zeta \int_{\partial\Omega} \left(|u_\mu|^{p-2}u_\mu - |v_\mu|^{p-2}v_\mu\right)(u_\mu - v_\mu)\, d\sigma,
\end{aligned} \tag{4.12}$$

while (2.2) entails

$$\int_{\partial\Omega} \left(|u_\mu|^{p-2}u_\mu - |v_\mu|^{p-2}v_\mu\right)(u_\mu - v_\mu)\, d\sigma \ge 2^{2-p}\|u_\mu - v_\mu\|_{p,\partial\Omega}^p. \tag{4.13}$$

Thus, from (4.12)–(4.13) it follows

$$\begin{aligned}
&\langle A_p(u_\mu) - A_p(v_\mu), u_\mu - v_\mu\rangle + \mu\langle A_2(u_\mu) - A_2(v_\mu), u_\mu - v_\mu\rangle \\
&\quad + \zeta \int_{\partial\Omega} \left(|u_\mu|^{p-2}u_\mu - |v_\mu|^{p-2}v_\mu\right)(u_\mu - v_\mu)\, d\sigma \\
&\ge 2^{2-p}\|u_\mu - v_\mu\|_{\zeta,p}^p + \mu\|\nabla(u_\mu - v_\mu)\|_2^2.
\end{aligned} \tag{4.14}$$

As in (a), by applying (U1), (U2), Hölder's inequality, and (3.14), we have for the right-hand side of (4.9)

$$\begin{aligned}
&\int_\Omega (f(x, u_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla u_\mu))(u_\mu - v_\mu)\, dx \\
&\quad + \int_\Omega (f(x, v_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla v_\mu))(u_\mu - v_\mu)\, dx \\
&\quad + \int_{\partial\Omega} (g(x, u_\mu) - g(x, v_\mu))(u_\mu - v_\mu)\, d\sigma \\
&\le c_1\|u_\mu - v_\mu\|_p^p + \int_\Omega \left(f\left(x, v_\mu, \nabla\left(\frac{1}{2}(u_\mu - v_\mu)^2\right)\right) - \rho(x)\right) dx \\
&\quad + c_2\|u_\mu - v_\mu\|_{p,\partial\Omega}^p - c_3\|u_\mu - v_\mu\|_{2,\partial\Omega}^2 \\
&\le \max\left\{c_1, \frac{c_2}{\zeta}\right\} \|u_\mu - v_\mu\|_{\zeta,p}^p + c_4 \int_\Omega |u_\mu - v_\mu||\nabla(u_\mu - v_\mu)|\, dx \\
&\quad - c_3\|u_\mu - v_\mu\|_{2,\partial\Omega}^2 \\
&\le \max\left\{c_1, \frac{c_2}{\zeta}\right\} \|u_\mu - v_\mu\|_{\zeta,p}^p + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}}\|u_\mu - v_\mu\|_{\zeta,2}^2 - c_3\|u_\mu - v_\mu\|_{2,\partial\Omega}^2.
\end{aligned} \tag{4.15}$$

Combining (4.9) with (4.14)–(4.15) yields

$$\begin{aligned}
&2^{2-p}\|u_\mu - v_\mu\|_{\zeta,p}^p + \mu\|\nabla(u_\mu - v_\mu)\|_2^2 \\
&\le \max\left\{c_1, \frac{c_2}{\zeta}\right\} \|u_\mu - v_\mu\|_{\zeta,p}^p + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}}\|u_\mu - v_\mu\|_{\zeta,2}^2 - c_3\|u_\mu - v_\mu\|_{2,\partial\Omega}^2,
\end{aligned}$$

which directly leads to

$$\begin{aligned} &2^{2-p}\|u_\mu - v_\mu\|_{\zeta,p}^p + \min\left\{\mu, \frac{c_3}{\zeta}\right\}\|u_\mu - v_\mu\|_{\zeta,2}^2 \\ &\leq \max\left\{c_1, \frac{c_2}{\zeta}\right\}\|u_\mu - v_\mu\|_{\zeta,p}^p + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}}\|u_\mu - v_\mu\|_{\zeta,2}^2. \end{aligned}$$

Therefore, if (4.8) is satisfied, then $u_\mu = v_\mu$.

References

[1] S.A. Marano, P. Winkert, On a quasilinear elliptic problem with convection term and nonlinear boundary condition, Nonlinear Anal. 187 (2019) 159–169.